ECE 313: Electromagnetic Waves

Lecture 4: Maxwell’s Equations

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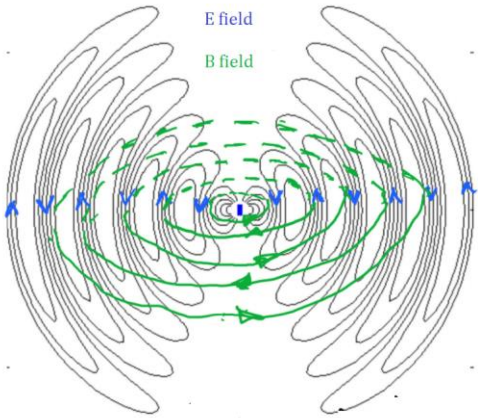
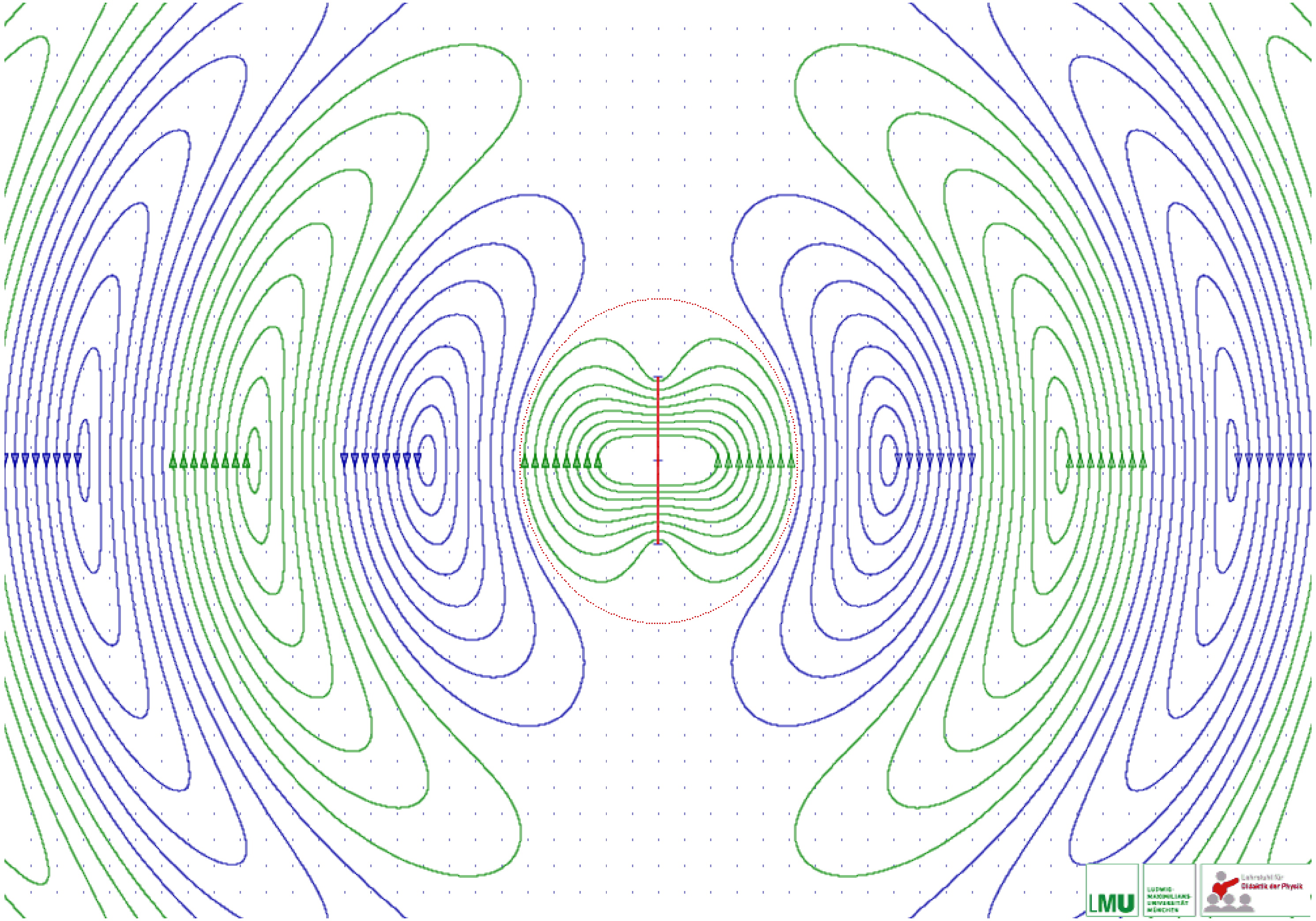
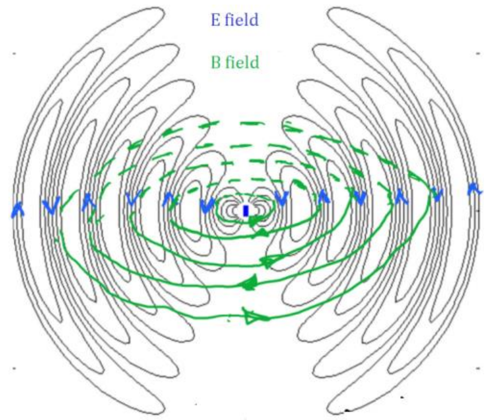
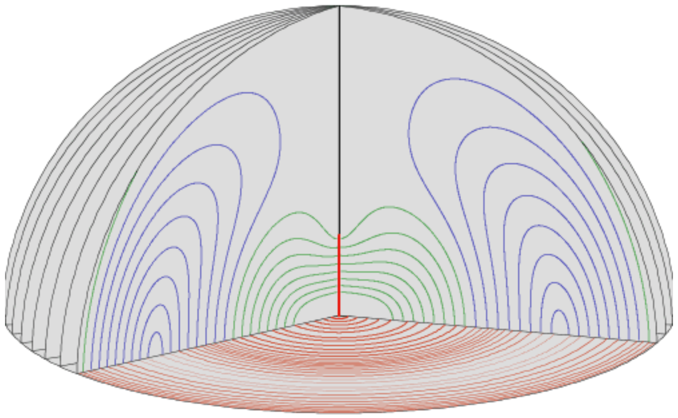
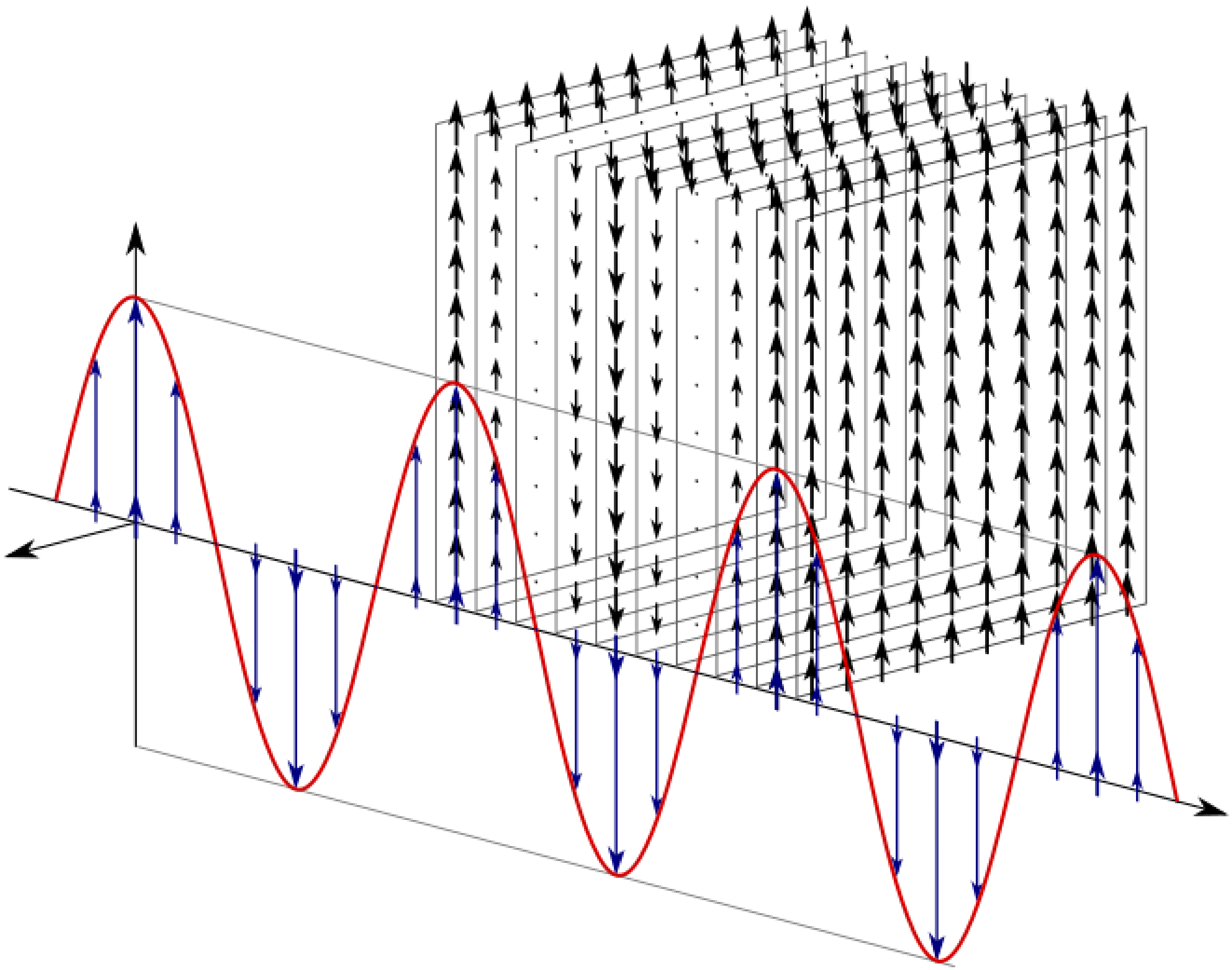


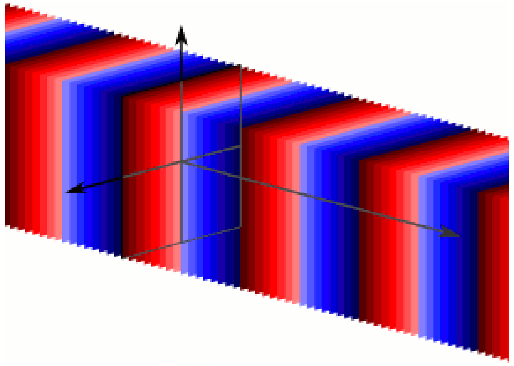
	Electrostatic	Magnetostatic	Maxwell's Equations	Time varying electromagnetics
Source	Static electric charges	Steady state current		Time varying currents
Equations	$\nabla . \bar{D} = \rho$ $\nabla \times \bar{E} = 0$	$\nabla . \bar{B} = 0$ $\nabla \times \bar{H} = \bar{J}$		$\nabla . \bar{D} = \rho$ $\nabla . \bar{B} = 0$ $\nabla \times \bar{E} = -\frac{\partial B}{\partial t}$ $\nabla \times \bar{H} = \bar{J} + \frac{\partial D}{\partial t}$ Term added by maxwell
Constitutive relation	D=εE	H=B/μ		Same
Characteristics	• E,D,B,H only fnc. Of space (x,y,z) • Not a function of time • Independently defined • Special form of Maxwell equations			• Function of both time and space • E and B are mutually dependent • A particular solution to Maxwell's equation :EM wave propagation

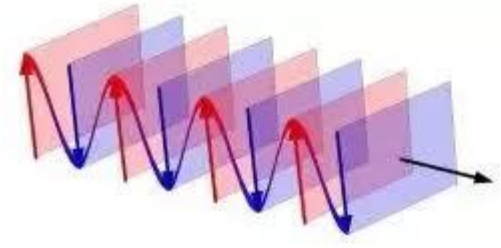



E field
B field

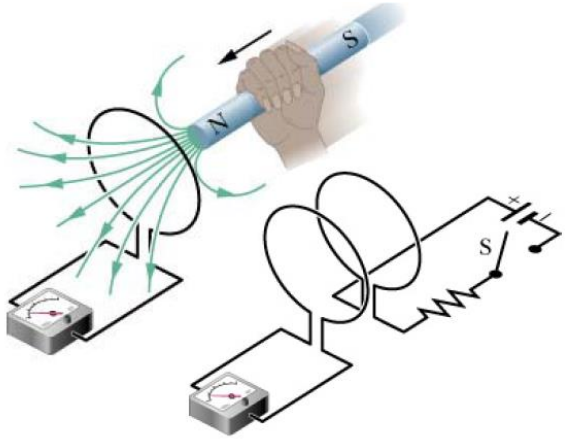








Faraday's law



- Faraday's experiment

a current was induced in a conducting loop when the magnetic flux linking the loop changes.

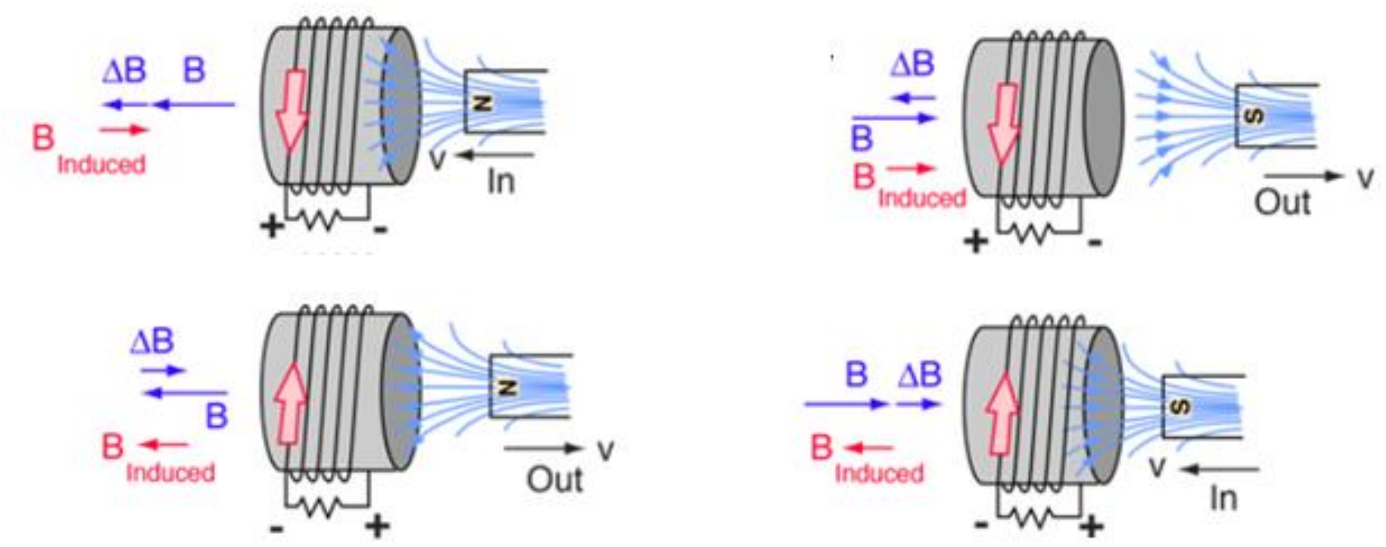
So Time varying magnetic field produces an emf which may establish a current in closed circuit. $emf = -\frac{d\varphi}{dt}\ v$

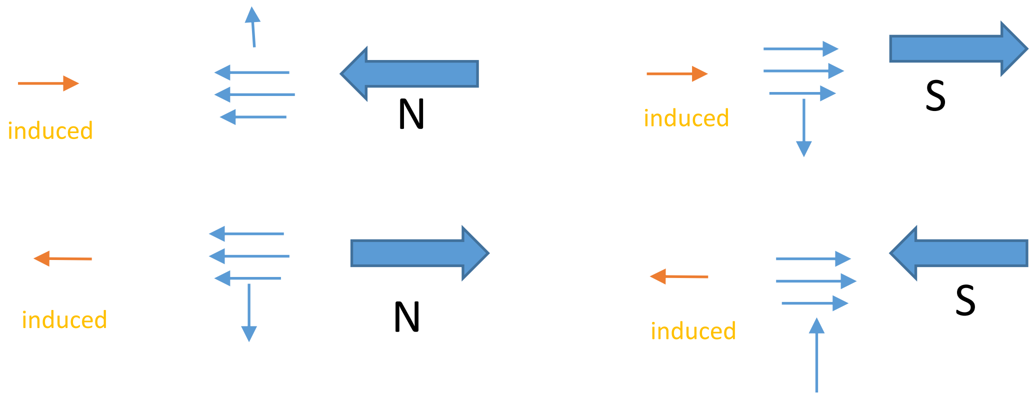
Lenz's law

φ: magnetic flux passes through surface whose perimeter is closed path

Lenz's law: Induced emf results in a current flowing in a direction *opposing the change of the linking magnetic flux*

Lenz's law: Induced emf results in a current flowing in a direction *opposing the change of the linking magnetic flux*





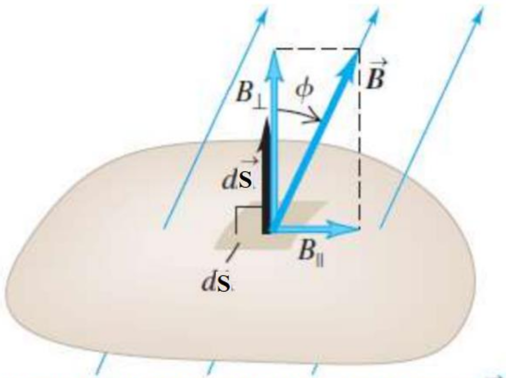
Faraday's law

- Fundamental postulate
- $v = -\frac{d\varphi}{dt} = -\frac{d}{dt}\int_s B.ds$
- $v = \oint_c E.dl$ →[Apply stokes theorem for curl E]

  $= \int_s \nabla \times E\, ds$
- $\int_s \nabla \times E\, ds = \int_s -\frac{\partial B}{\partial t}$ ds
- $\nabla \times E = -\frac{\partial B}{\partial t}$

v :emf induced in circuit with contour C (v)

φ : magnetic flux crossing surface S (Wb)



Magnetic flux through element of area $d\vec{s}$

$d\Phi_B = \vec{B}\cdot d\vec{s} = B_\perp ds = B\, ds \cos\phi$

FARADAY LAW OF ELECTROMAGNETIC INDUCTION: the electromotive force in a stationary closed circuit is equal to the negative rate of increase of magnetic flux linking the circuit

Example

EXAMPLE 7–1 A circular loop of N turns of conducting wire lies in the xy-plane with its center at the origin of a magnetic field specified by $\mathbf{B} = \mathbf{a}_z B_0 \cos(\pi r/2b) \sin \omega t$, where b is the radius of the loop and ω is the angular frequency. Find the emf induced in the loop.

Solution The problem specifies a stationary loop in a time-varying magnetic field; hence Eq. (7–6) can be used directly to find the induced emf, $\mathscr{V}$. The magnetic flux linking each turn of the circular loop is

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s}$$

$$= \int_0^b \left[\mathbf{a}_z B_0 \cos \frac{\pi r}{2b} \sin \omega t \right] \cdot (\mathbf{a}_z 2\pi r\, dr)$$

$$= \frac{8b^2}{\pi} \left(\frac{\pi}{2} - 1 \right) B_0 \sin \omega t.$$

$$\int u \cos(u) du = \cos u + u \sin u$$

$$u = \frac{\pi r}{2b} \rightarrow du = \frac{\pi}{2b} dr$$

Since there are N turns, the total flux linkage is $N\Phi$, and we obtain

$$\mathscr{V} = -N \frac{d\Phi}{dt}$$

$$= -\frac{8N}{\pi} b^2 \left(\frac{\pi}{2} - 1 \right) B_0 \omega \cos \omega t \qquad \text{(V)}.$$

The induced emf is seen to be 90° out of time phase with the magnetic flux. ■

[David_K._Cheng]_Field_and_Wave_Electromagnetics.

Example

assume a simple magnetic field which increases exponentially with time within the cylindrical region $\rho < b$,

$$\mathbf{B} = B_0 e^{kt} \mathbf{a}_z$$

Compute induced emf and E

where $B_0 =$ constant. Choosing the circular path $\rho = a$, $a < b$, in the $z = 0$ plane,

Get E from

$$\text{emf} = \oint_c E.dl = -\frac{d}{dt}\int_s B.ds$$

$$\text{emf} = 2\pi a E_\phi = -kB_0 e^{kt} \pi a^2$$

If we now replace a by ρ, $\rho < b$, the electric field intensity at any point is

$$\mathbf{E} = -\tfrac{1}{2} k B_0 e^{kt} \rho \mathbf{a}_\phi$$

Or Get E from

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

Let us now attempt to obtain the same answer from (6), which becomes

$$(\nabla \times \mathbf{E})_z = -kB_0 e^{kt} = \frac{1}{\rho}\frac{\partial(\rho E_\phi)}{\partial \rho}$$

Multiplying by ρ and integrating from 0 to ρ (treating t as a constant, since the derivative is a partial derivative),

$$-\tfrac{1}{2} k B_0 e^{kt} \rho^2 = \rho E_\phi$$

or

$$\mathbf{E} = -\tfrac{1}{2} k B_0 e^{kt} \rho \mathbf{a}_\phi$$

Continuity equation

For net current to come out closed surface there must be A decrease in positive charges within it

$$I = \oint_s \underline{J}_c . \underline{ds} = -\frac{dQ}{dt} = -\frac{d}{dt}\int_v \rho_v dv$$

$$\oint_s \underline{J}_c . \underline{ds} = \int_v \nabla . \underline{J}_c dv$$

Divergence theorem

$$\int_v \nabla . \underline{J}_c dv = -\frac{d}{dt}\int_v \rho_v dv$$

$$\nabla . \underline{J}_c = -\frac{d\rho_v}{dt}$$

ρ_v

Kirchoff’s current law

In the steady state, the charge (or current) flowing into any point in the circuit has to be equal the charge (or current) flowing out.

$$for\ steady\ state\ current\ (\frac{d\rho_v}{dt} = 0)$$

$$\nabla . \underline{J}_c = 0$$

J_c : conduction current density

Displacement current

Ampere's law for magnetostatic: $\nabla \times \bar{H} = \bar{J}$ since div curl=0

$$\nabla . \nabla \times \bar{H} = \nabla . \bar{J} = 0$$

But continuity of current $\nabla . \bar{J} = -\frac{\partial \rho_v}{\partial t} \neq 0$ (time varying)

So we must add term for Ampere's law in time varying

$$\nabla \times \bar{H} = \bar{J} + \bar{J}_d$$

$\nabla . \nabla \times \bar{H} = \nabla . \bar{J} + \nabla . \bar{J}_d = 0$ $->$ $\nabla . \bar{J} = -\nabla . \bar{J}_d$

$\nabla . \bar{J}_d = \frac{\partial \rho_v}{\partial t}$ but $\nabla . \bar{D} = \rho_v$

$\nabla . \bar{J}_d = \frac{\partial \nabla . \bar{D}}{\partial t}$ → $\bar{J}_d == \frac{\partial \bar{D}}{\partial t}$

$\bar{J}_d$:is the displacement current

$\bar{J}$: is conduction current density($\bar{J} = \sigma \bar{E}$)

$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$ Ampere's law for time varying field

Example

EXAMPLE 7–5 An a-c voltage source of amplitude V_0 and angular frequency ω, $v_c = V_0 \sin \omega t$, is connected across a parallel-plate capacitor C_1, as shown in Fig. 7–7. (a) Verify that the displacement current in the capacitor is the same as the conduction current in the wires. (b) Determine the magnetic field intensity at a distance r from the wire.

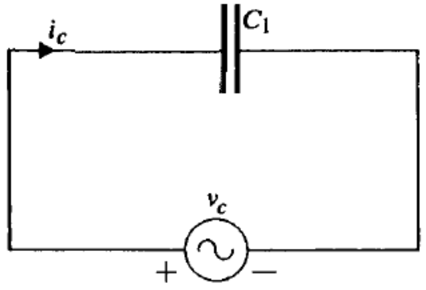


FIGURE 7–7

Solution

a) The conduction current in the connecting wire is

$$i_C = C_1 \frac{dv_C}{dt} = C_1 V_0 \omega \cos \omega t \qquad \text{(A)}.$$

For a parallel-plate capacitor with an area A, plate separation d, and a dielectric medium of permittivity ϵ the capacitance is

$$C_1 = \epsilon \frac{A}{d}.$$

With a voltage v_C appearing between the plates, the uniform electric field intensity E in the dielectric is equal to (neglecting fringing effects) $E = v_C/d$, whence

$$D = \epsilon E = \epsilon \frac{V_0}{d} \sin \omega t.$$

The displacement current is then

$$i_D = \int_A \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s} = \left(\epsilon \frac{A}{d}\right) V_0 \omega \cos \omega t$$

$$= C_1 V_0 \omega \cos \omega t = i_C.$$

b) The magnetic field intensity at a distance r from the conducting wire can be found by applying the generalized Ampère's circuital law

C in Fig. 7–7. Two typical open surfaces with rim C may be chosen: (1) a planar disk surface S_1, or (2) a curved surface S_2 passing through the dielectric medium. Symmetry around the wire ensures a constant H_ϕ along the contour C. The line integral on the left side of Eq. (7–54b) is

$$\oint_C \mathbf{H} \cdot d\boldsymbol{\ell} = 2\pi r H_\phi.$$

: no charges are deposited along the wire and, consequently, $\mathbf{D} = 0$.

$$\int_{S_1} \mathbf{J} \cdot d\mathbf{s} = i_C = C_1 V_0 \omega \cos \omega t.$$

Since the surface S_2 passes through the dielectric medium, no conduction current flows through S_2. If the second surface integral were not there, the right side of Eq. (7–54b) would be zero. This would result in a contradiction. The inclusion of the displacement-current term by Maxwell eliminates this contradiction. As we have shown in part (a), $i_D = i_C$. Hence we obtain the same result whether surface S_1 or surface S_2 is chosen. Equating the two previous integrals, we find that

$$H_\phi = \frac{C_1 V_0}{2\pi r} \omega \cos \omega t \qquad \text{(A/m)}.$$